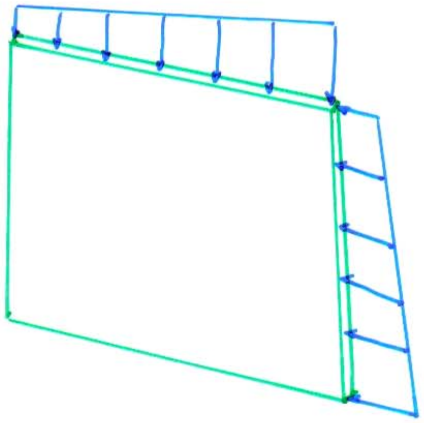
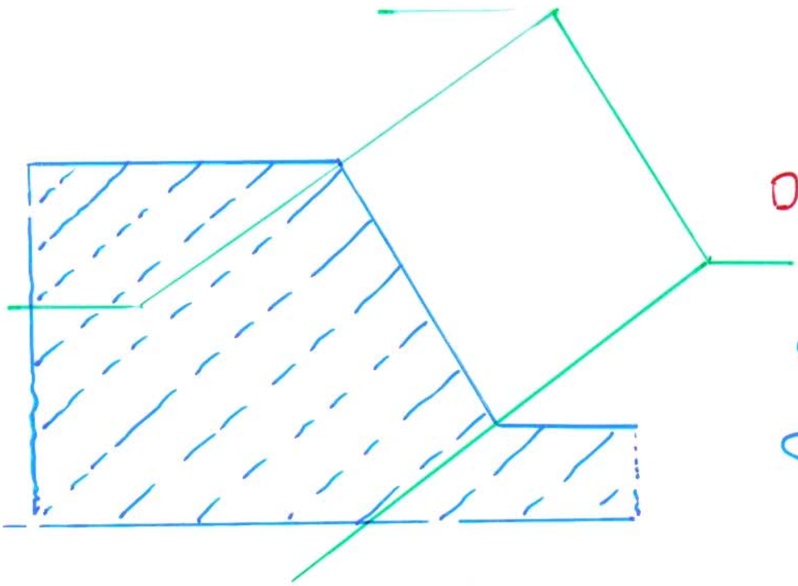
TEMA 6 - TENSIONES EN ROCA

TENSIONES EN EL PLANO



TENSION PLANA

$\sigma_2 = 0$

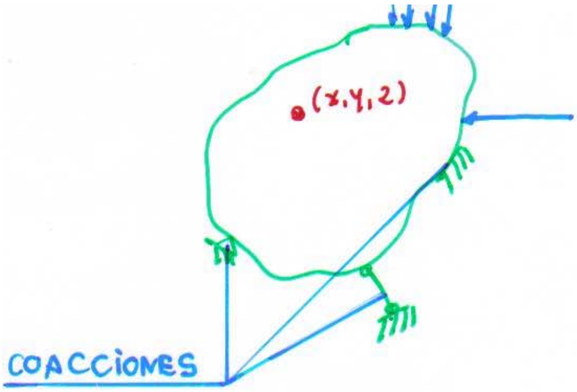


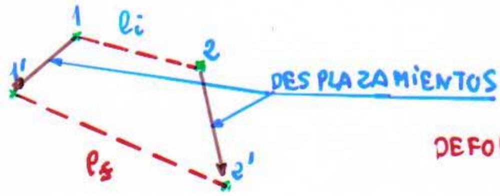
DEFORMACION PLANA

$\sigma_2 = f(\sigma_1 ; \sigma_3)$

$\sigma_1 > \sigma_2 > \sigma_3$

ESTADO MECANICO DEL SOLIDO

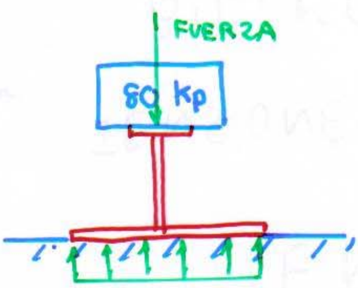




DEFORMACION $\varepsilon = \lim \dfrac{l_f - l_i}{l_i}$

(ESCALAR)

si $2 \rightarrow 1 \rightarrow l_i \rightarrow 0$ $\quad \vec{\varepsilon}_{ij} = \dfrac{\partial \vec{d}_i}{\partial x_j}$ ¡ VECTORIAL !

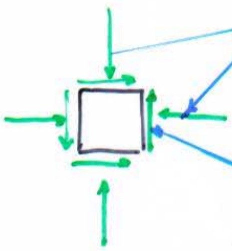


TENSION = FUERZA / AREA

$$\vec{\sigma} = \lim_{A \to 0} \frac{\vec{F}}{A} = \frac{d\vec{F}}{dA}$$

$\vec{F}$: Vector $\quad A$: escalar $\Rightarrow \vec{\sigma}$: VECTORIAL

CRITERIO DE SIGNOS



FUERZAS NORMALES

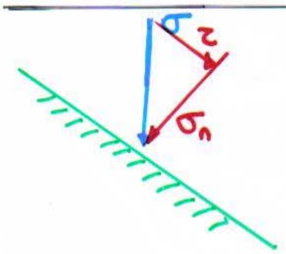
\+ COMPRESION.

FUERZAS TANGENCIALES

\+ 

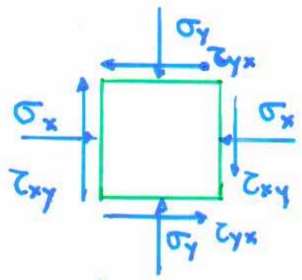
CONTRA HORARIO

DESCOMPOSICIÓN DE TENSIÓN



σ_n : TENSION NORMAL
σ : TENSION TOTAL
τ_{xy} : TENSION TANGENCIAL

EL ESTADO DE TENSIONES EN UN PUNTO QUEDA DEFINIDO POR LOS VALORES EN DOS PLANOS PERPENDICULARES

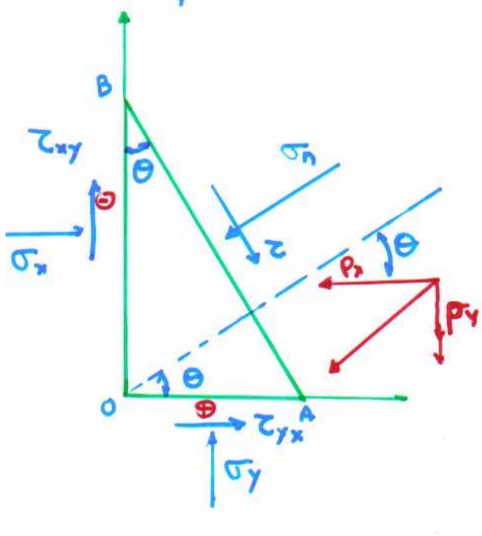


EQUILIBRIO MOMENTOS $\tau_{yx} = \tau_{xy}$

τ_{xy}

y: EJE AL QUE ES PARALELO

x: PLANO "x" = CTe. EN QUE SE CONTIENE



ESTABLECIENDO EQUILIBRIO

$p_x \cdot AB = \sigma_x \cdot OB + \tau_{yx} \cdot OA$

$OB = AB \cdot \cos\theta$

$OA = AB \cdot \sin\theta$

$\Downarrow$

$p_x = \sigma_x \cdot \cos\theta + \tau_{yx} \cdot \text{sen}\,\theta$

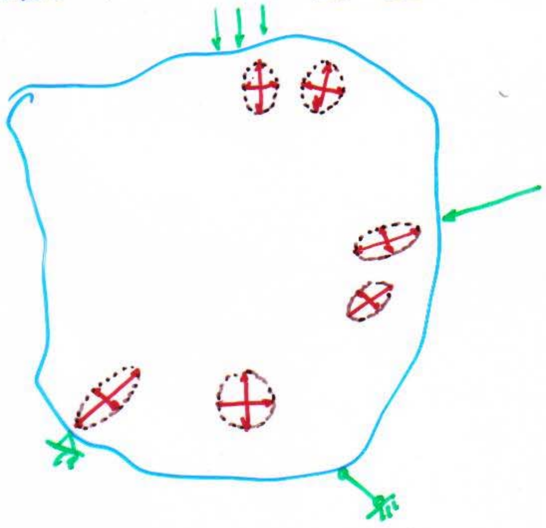
$p_y = \sigma_y \cdot \text{sen}\,\theta + \tau_{xy} \cdot \cos\theta$

Si $\tau_{xy} = \tau_{yx} = \emptyset$ $\quad$ $\sigma_x = \sigma_1$ $\quad$ $p_x = \sigma_1 \cdot \cos\theta$

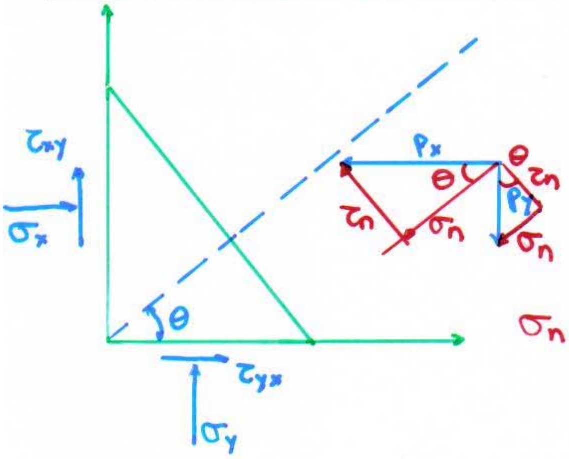
$\sigma_y = \sigma_3$ $\quad$ $p_y = \sigma_3 \cdot \sin\theta$

$\frac{p_x^2}{\sigma_1^2} + \frac{p_y^2}{\sigma_3^2} = 1 \Rightarrow$ ELIPSES DE TENSIONES

LOS TENSIONES EN EL PLANO → ELIPSES DE TENSIONES



CIRCULO DE MOHR. ISOSTATICAS



$$\sigma_n = p_x \cdot \cos\theta + p_y \operatorname{sen}\theta$$

$$\tau = -p_x \cdot \sin\theta + p_y \cos\theta$$

$$\sigma_n = \sigma_x \cos^2\theta + 2\tau_{xy} \sin\theta\cos\theta + \sigma_y \sin^2\theta$$

$$\Downarrow$$

$$\sigma_n = \frac{1}{2}(\sigma_x + \sigma_y) + \cos 2\theta \, \frac{1}{2}(\sigma_x - \sigma_y) + \tau_{xy} \operatorname{sen} 2\theta$$

$$\tau = (\sigma_y - \sigma_x) \operatorname{sen}\theta\cos\theta + \tau_{xy}(\cos^2\theta - \operatorname{sen}^2\theta)$$

$$\tau = \frac{1}{2}(\sigma_y - \sigma_x)\operatorname{sen} 2\theta + \tau_{xy}\cos 2\theta = \cos 2\theta\left[(\sigma_y - \sigma_x)\frac{1}{2} \times \operatorname{tg} 2\theta + \tau_{xy}\right]$$

SI HACEMOS $\tau = 0 \Rightarrow 0 = \frac{1}{2}(\sigma_y - \sigma_x) \cdot Tg\, 2\theta + \tau_{xy}$

ECUACION ISOSTATICAS

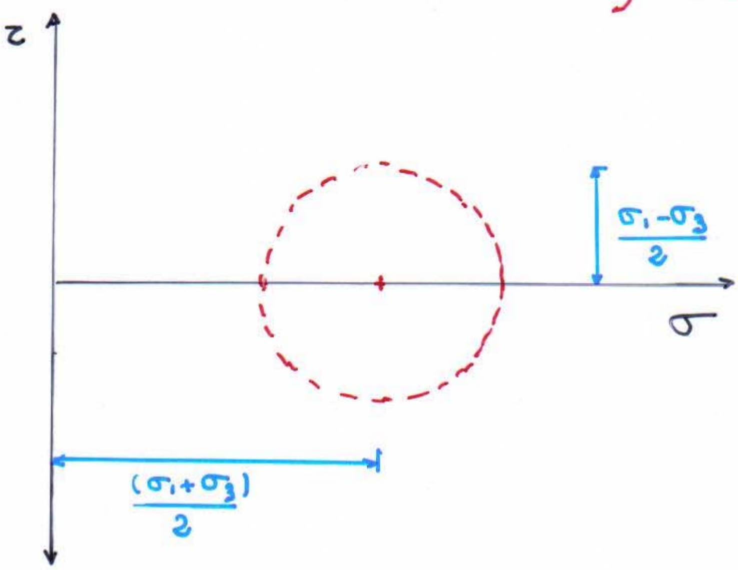
SI HACEMOS $\sigma_y = \sigma_3$ $\quad \sigma_x = \sigma_1$ $\quad \tau_{xy} = 0$

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\,\text{sen}\, 2\theta$$

$\Downarrow$

$$\left.\begin{array}{ll} \text{CIRCULO} \quad \text{CENTRO}: & \frac{1}{2}(\sigma_1 + \sigma_3)\ ,\ 0 \\ \text{RADIO} & \frac{1}{2}(\sigma_1 - \sigma_3) \end{array}\right\} \text{CIRCULO DE MOHR.}$$



CIRCULO DE MOHR ⟷ SISTEMA GRAFICO DE OBTENCION DE TENSIONES EN CUALQUIER DIRECCION DE CASO PLANO.

↓

INDEPENDIENTE CRITERIO DE ROTURA

ISOSTATICAS

Se tenía $\quad \tau = \cos 2\theta \cdot \left[\frac{1}{2} (\sigma_y - \sigma_x) \cdot \operatorname{tg} 2\theta + \tau_{xy} \right]$

HACIENDO $\quad \tau = 0 \Rightarrow \operatorname{tg} 2\theta = \dfrac{2 \tau_{xy}}{\sigma_y - \sigma_x}$

$$\operatorname{Tg} 2\theta = \frac{2 \operatorname{tg} \theta}{1 - \operatorname{Tg}^2 \theta} = \frac{2 \frac{dy}{dx}}{1 - \left(\frac{dy}{dx} \right)^2} = \frac{2 \tau_{xy}}{\sigma_y - \sigma_x}$$

$$\Downarrow$$

$$\left[\frac{dy}{dx} \right]_1^2 = - \frac{\sigma_x - \sigma_y}{2 \tau_{xy}} \overset{2}{\underset{1}{\pm}} \sqrt{1 + \frac{(\sigma_x - \sigma_y)^2}{(2 \tau_{xy})^2}}$$

ECUACION DIFERENCIAL DE LAS ISOSTATICAS

$$\left[\frac{dy}{dx} \right]_1 \times \left[\frac{dy}{dx} \right]^2 = -1 \quad \Rightarrow \quad \text{PERPENDICULARES}$$