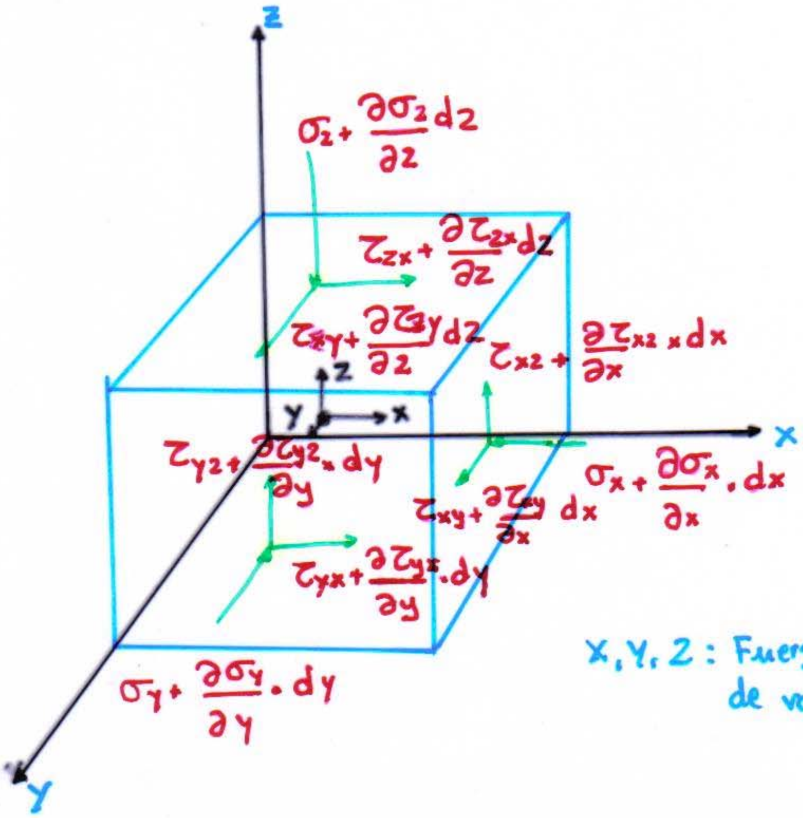
TENSIONES Y DEFORMACIONES ROCAS



X, Y, Z : Fuerzas por unidad de volumen

ESTABLECIENDO EQUILIBRIO DE FUERZAS ⇒

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial x} + X = 0$$

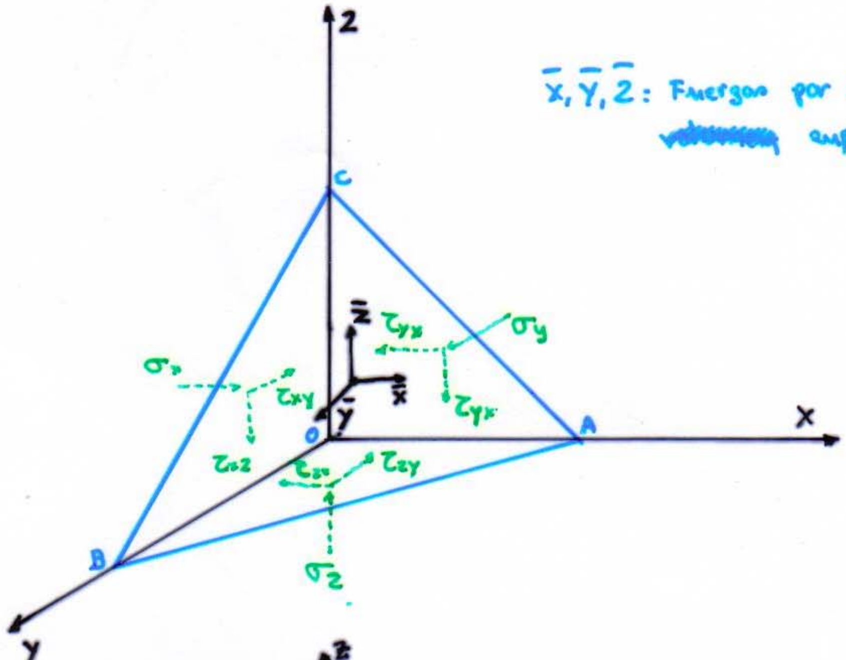
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + Y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Z = 0$$

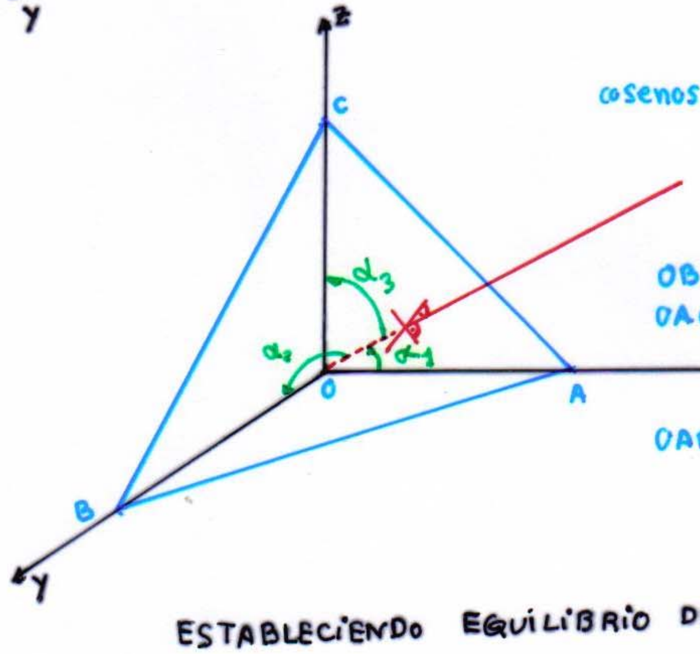
Ecuaciones de equilibrio interno

Si: $S_{ii} = \sigma$ y $S_{ij} = \tau$. $\rightarrow \sum_{i=1}^{3} \frac{\partial S_{ij}}{\partial x_j} + X_{ij} = 0$

Tomando momentos $\tau_{xy} = \tau_{yx}$ . . . . $S_{ij} = S_{ji}$



$\bar{X}, \bar{Y}, \bar{Z}$: Fuerzas por unidad de superficie.



cosenos directores $\begin{cases} l = \cos \alpha_1 \\ m = \cos \alpha_2 \\ n = \cos \alpha_3 \end{cases}$

$OBC = ABC \cdot \cos \alpha_1 = ABC \cdot l$

$OAC = ABC \cdot \cos \alpha_2 = ABC \cdot m$

$OAB = ABC \cdot \cos \alpha_3 = ABC \cdot n$

ESTABLECIENDO EQUILIBRIO DE FUERZAS

$$\sigma_x \cdot l + \tau_{xy} \cdot m + \tau_{xz} \cdot n = \bar{X}$$

$$\tau_{xy} \cdot l + \sigma_y \cdot m + \tau_{yx} \cdot n = \bar{Y}$$

$$\tau_{xz} \cdot l + \tau_{yz} \cdot m + \sigma_z \cdot n = \bar{Z}$$

$\bar{X}, \bar{Y}, \bar{Z}$ $\begin{cases} \text{Fuerzas en contorno} \\ \text{Tensiones resultantes en plano} \end{cases}$

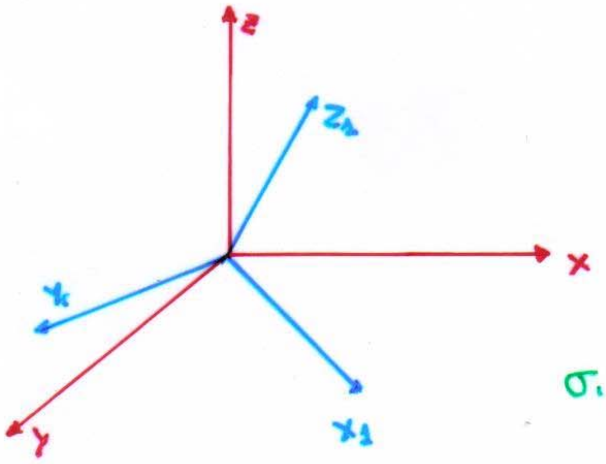
$$\begin{Bmatrix} X\\ Y\\ Z\end{Bmatrix} = \begin{Bmatrix} p_{nx}\\ p_{ny}\\ p_{nz}\end{Bmatrix} = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z\end{vmatrix} \cdot \begin{Bmatrix} l\\ m\\ n\end{Bmatrix}$$

TENSOR DE TENSIONES

MATRIZ SIMETRICA $\Rightarrow$ DEFINIDA POSITIVA ($\Delta > 0$ POR DISIPACION ENERGIA) $\rightarrow$ PUEDE DIAGONALIZARSE SEGUN UNA RELACION:

$$\sigma = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z\end{vmatrix} \qquad \sigma_1 = \begin{vmatrix} \sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3\end{vmatrix}$$

$$\sigma_1 = A \cdot \sigma \cdot A^T \quad y \quad A = \begin{vmatrix} \cos(x_1,x) & \cos(x_1,y) & \cos(x_1,z)\\ \cos(y_1,x) & \cos(y_1,y) & \cos(y_1,z) \\ \cos(z_1,x) & \cos(z_1,y) & \cos(z_1,z)\end{vmatrix}$$



$\sigma_1, \sigma_2, \sigma_3$: Autovalores

$$\text{Det.} \begin{vmatrix} \sigma_x - \sigma_i & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_y - \sigma_i & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z - \sigma_i\end{vmatrix} = \emptyset$$

ECUACION DE 3er GRAD

RESOLVIENDO EL DETERMINANTE

$$\sigma_i^3 - I_1 \sigma_i^2 + I_2 \sigma_i - I_3 = 0$$

ECUACION CARACTERISTICA

$$\Downarrow$$

$$\sigma_1 \quad \sigma_2 \quad \sigma_3$$

INVARIANTES

$$\begin{cases} I_1 = \sigma_x + \sigma_y + \sigma_z \\ I_2 = \sigma_x \sigma_y + \sigma_y \cdot \sigma_z + \sigma_z \cdot \sigma_x - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{zy}^2 \\ I_3 = \sigma_x \cdot \sigma_y \cdot \sigma_z - \sigma_x \tau_{zy}^2 - \sigma_y \cdot \tau_{xz}^2 - \sigma_z \tau_{xy}^2 - 2 \tau_{xy} \tau_{yz} \tau_{xz} \end{cases}$$

EN FUNCION DE LAS TENSIONES PRINCIPALES

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \sigma_1 \cdot \sigma_2 + \sigma_2 \cdot \sigma_3 + \sigma_1 \cdot \sigma_3$$

$$I_3 = \sigma_1 \cdot \sigma_2 \cdot \sigma_3$$

PLANO OCTAHEDRICO

$$p = \sigma_{oct} = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) = \frac{1}{3} \cdot I_1$$

ANGULO DE $\quad arc.\cos \sqrt{\frac{1}{3}}$ CON DIRECCION PRINCIPAL

$$\tau_{oct} = \sqrt{\frac{1}{3} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]} = \pm \sqrt{\frac{2}{9} \left[I_1^2 - 3 I_2 \right]}$$

Si $\sigma_2 = \sigma_3 < \sigma_1$, resulta

$$p = \frac{1}{3} (\sigma_1 + 2\sigma_3)$$

$$\tau_{oct} = (\sigma_1 - \sigma_3) \cdot \sqrt{\frac{2}{3}}$$

TENSION VOLUMETRICA Y DESVIADORA

$$\sigma = \begin{vmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{vmatrix} + \begin{bmatrix} \sigma_x - p & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - p & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - s \end{bmatrix}$$

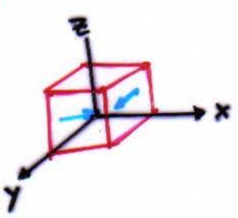
$\downarrow$ TENSOR VOLUMETRICO (VARIACION VOLUMEN)

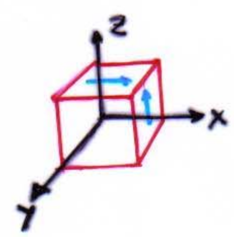
$\downarrow$ TENSOR DESVIADOR (VARIACION DE FORMA)

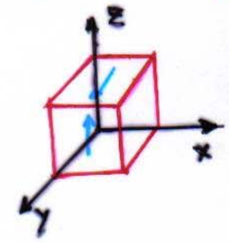
LLAMANDO:

$S_x = \sigma_x - p \qquad S_y = \sigma_y - p \qquad S_z = \sigma_z - p$

$$\begin{vmatrix} S_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & S_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & S_z \end{vmatrix} = \begin{vmatrix} 0 & \tau_{xy} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & \tau_{xz} \\ 0 & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau_{yz} \\ 0 & \tau_{yz} & 0 \end{vmatrix} +$$







$$+ \begin{vmatrix} S_x & 0 & 0 \\ 0 & -S_x & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 \\ 0 & -S_z & 0 \\ 0 & 0 & S_z \end{vmatrix}$$