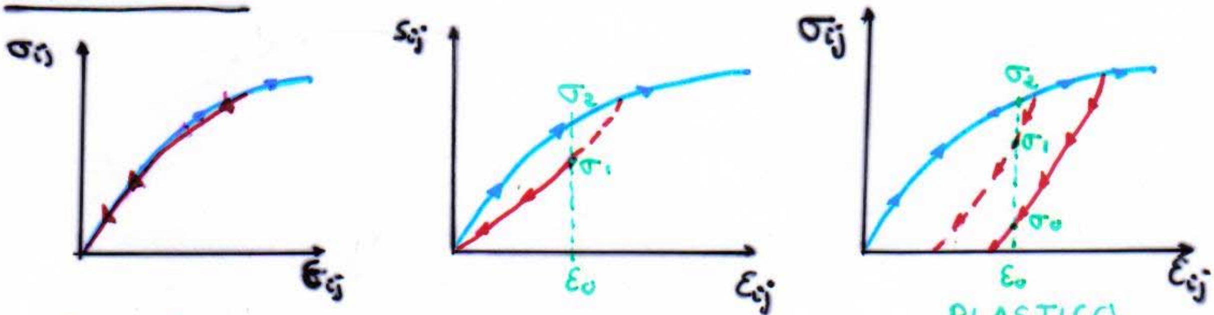
ELASTICIDAD



ELASTICO PLASTICO. PLASTICO

MATERIAL ELASTICO $\Leftrightarrow$ HAY UNA CORRESPONDENCIA BIUNIVOCA ENTRE DEFORMACIONES Y TENSIONES

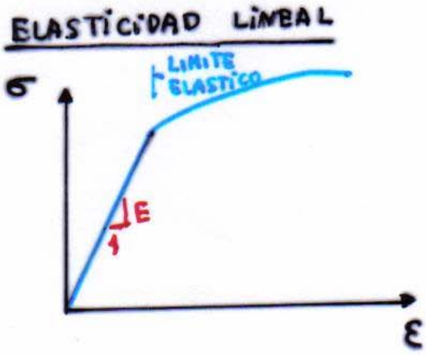
$\Downarrow$

CONOCIDAS DEFORMACIONES

$\Downarrow$

PUEDEN OBTENERSE TENSIONES $\rightarrow$ INDEPENDIENTE DE HISTORIA.

ELASTICIDAD LINEAL



$$\varepsilon = \frac{\sigma}{E}$$

E: Modulo de Young.

LAS FORMULAS GENERALES SON:

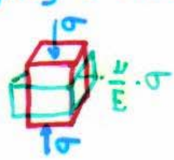
$$\varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_y = -\frac{\nu}{E}\sigma_x + \frac{1}{E}\sigma_y - \frac{\nu}{E}\sigma_z \qquad \gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\varepsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{1}{E}\sigma_z \qquad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

$G = \dfrac{E}{2(1+\nu)}$. Modulo de rigidez o elasticidad transversal

ν: Coeficiente de Poisson



Si llamamos:

$e = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

SE OBTIENE:

$$\left.\begin{aligned}
\sigma_x &= \lambda e + 2G\,\varepsilon_x \\
\sigma_y &= \lambda e + 2G\,\varepsilon_y \\
\sigma_z &= \lambda e + 2G\,\varepsilon_z \\
\tau_{xy} &= G \cdot \gamma_{xy} \\
\tau_{yz} &= G \cdot \gamma_{yz} \\
\tau_{xz} &= G \cdot \gamma_{xz}
\end{aligned}\right\}$$

ECUACIONES DE LAMÉ

λ, G : CONSTANTES DE LAMÉ

SUMANDO LAS TRES PRIMERAS ECUACIONES:

$(3\lambda + 2G) \cdot e = \sigma_x + \sigma_y + \sigma_z = E_1$ (1er invariante en tensiones)

$\Downarrow$

DILATACIÓN CÚBICA O ESFÉRICA $\Leftarrow$ CONSTANTE $\Rightarrow$ TENSIÓN OCTAÉDRICA CONSTANTE

MÓDULO DE COMPRESIÓN. $K' = 3\lambda + 2G = \dfrac{E}{1-2\nu}$

MÓDULO VOLUMÉTRICO, DE BULBO O ISOTRÓPICO $k = \dfrac{E}{3(1-2\nu)} = \dfrac{k'}{3}$

$\varepsilon_{vol} = \dfrac{p'}{k}$. p': OCTAÉDRICA

$\epsilon_s = \dfrac{q'}{3G}$ q': OCTAÉDRICA

EN FORMA DIFERENCIAL

$$d\varepsilon_{vol} = \frac{dp'}{k'} + 0 \cdot dq'$$

$$d\epsilon_s = 0 \cdot dp' + \frac{dq'}{3G}$$

Si $\nu = 0{,}5 \Rightarrow k = \dfrac{E}{3(1-2\nu)} \to \infty$ y $\varepsilon_{vol} = 0$ INDEPENDIENTE DE TENSIÓN

EN MATERIAL INCOMPRESIBLE $\nu = 0{,}50$

PROBLEMA. (TRIAXIAL)

$\sigma_2 = \sigma_3 = 100$ kPa

$\sigma_1 = 300$ kPa

$\varepsilon_1 = 6\%$ $\quad \varepsilon_2 = \varepsilon_3 = -1\%$.

OBTENER ε_{vol}, ε_s, E, ν, G y k.

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3 = 0{,}06 + 2 \times (-0{,}01) = 0{,}04 \Rightarrow 4\%$$

$$\varepsilon_s = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) = \frac{2}{3}(0{,}06 + 0{,}01) = \frac{0{,}14}{3} = 0{,}0466 \rightarrow 4{,}67\%$$

$$\left.\begin{array}{l} 0{,}06 = \dfrac{300}{E} - \dfrac{\nu}{E}(100 + 100) \\ -0{,}01 = \dfrac{100}{E} - \dfrac{\nu}{E}(300 + 100) \end{array}\right\} \quad \begin{array}{l} E = 3800 \text{ kPa} \\ \nu = 0{,}35 \end{array}$$

PROBLEMA (TRIAXIAL)

$\sigma_1 = 280$ kPa. $\quad \sigma_2 = \sigma_3 = 0$

$\varepsilon_1 = 6\%$ $\quad \varepsilon_2 = \varepsilon_3 = -1{,}5\%$

OBTENER E, ν, G y k.

$$\left.\begin{array}{l} 0{,}06 = \dfrac{280}{E} - \dfrac{\nu}{E}(0 + 0) \\ -0{,}015 = \dfrac{0}{E} - \dfrac{\nu}{E}(280 + 0) \end{array}\right\} \quad \begin{array}{l} E = 4667 \text{ kPa} \\ \nu = 0{,}25 \end{array}$$

$$k = \frac{E}{3(1 - 2\nu)} = \frac{4667}{3(1 - 2 \times 0{,}25)} = 3111 \text{ kPa}$$

$$G = \frac{E}{2 \times (1 + \nu)} = \frac{4667}{2 \times (1 + 0{,}25)} = 1867 \text{ kPa}$$

PROBLEMA (TRIAXIAL)

Se mantiene volumen constante $\quad \varepsilon_{vol} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$

$\varepsilon_2 = \varepsilon_3$

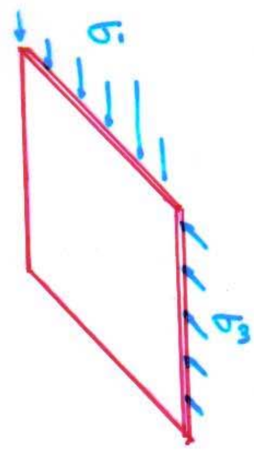
$\varepsilon_1 = 5\%$.

OBTENER ε_2, ε_3, E, ν, G y k

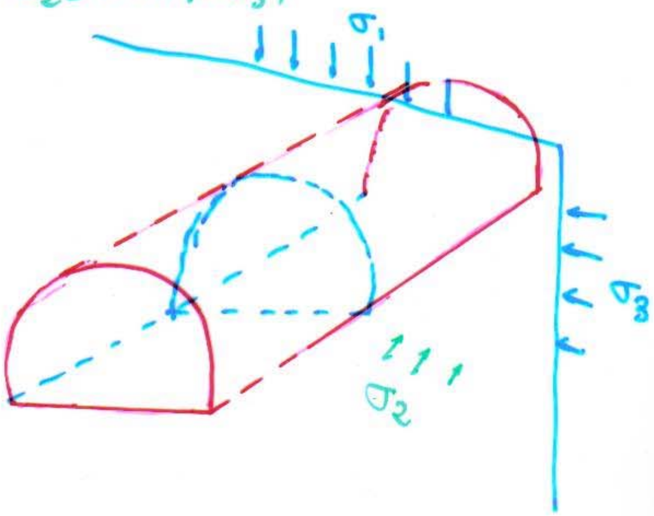
CASO PLANO.

En tension plana $\qquad \sigma_2 = 0 \qquad \varepsilon_2 = \frac{\nu}{E}(\sigma_1 + \sigma_3)$

DEFORMACION PLANA $\qquad \varepsilon_2 = 0 \qquad \sigma_2 = \nu(\sigma_1 + \sigma_3)$

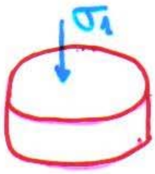


TENSIÓN PLANA



DEFORMACION PLANA.

EDOMETRO O COMPRESION EDOMETRICA



$\varepsilon_2 = \varepsilon_3 = \emptyset$ (celula)

$$\varepsilon_1 = \frac{\sigma_1}{D} \qquad D = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$\sigma_2 = \sigma_3 = \frac{\nu}{1-\nu} \cdot \sigma_1$$

OBTENCION PARAMETROS ELASTICOS

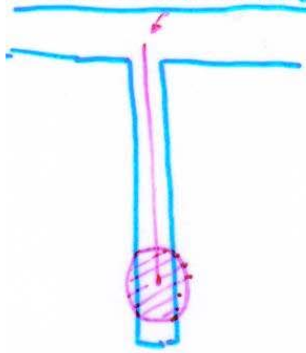
MODULO DE ELASTICIDAD E

Ensayo de compresión en prensa con medida de deformaciones

$\sigma_1, \ \varepsilon_1 \ \rightarrow \ \sigma_2 = \sigma_3 = 0. \qquad E = \frac{\sigma_1}{\varepsilon_1}$

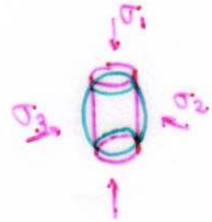
Si medimos ademas $\ \varepsilon_r \ \rightarrow \ \nu = \frac{\varepsilon_r}{\varepsilon_1}$

ENSAYO DILATOMETRICO



$P_r, \varepsilon_r \rightarrow E$

ENSAYO TRIAXIAL CON MEDIDA DEFORMACIONES



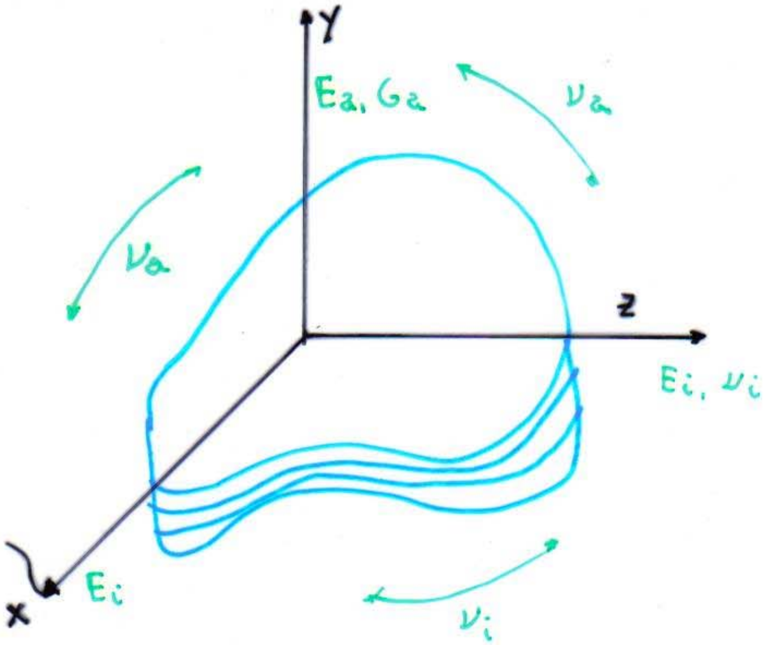
$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{2\nu}{E}\sigma_3$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \frac{\nu}{E}(\sigma_1 + \sigma_3)$$

EL VALOR DE "E" PUEDE VARIAR CON σ^3.

BIBLIOGRAFIA Y TABLAS

TERRENO ANISOTROPO



$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xz} \\ \gamma_{xy} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_i} & -\frac{\nu_a}{E_a} & -\frac{\nu_i}{E_i} & 0 & 0 & 0 \\ -\frac{\nu_a}{E_a} & \frac{1}{E_a} & -\frac{\nu_a}{E_a} & 0 & 0 & 0 \\ -\frac{\nu_i}{E_i} & -\frac{\nu_a}{E_a} & \frac{1}{E_i} & 0 & 0 & 0 \\ & & & \frac{2(1+\nu_i)}{E_i} & 0 & 0 \\ & & & & \frac{1}{G_a} & 0 \\ & & & & & \frac{1}{G_a} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xz} \\ \tau_{xy} \\ \tau_{yz} \end{Bmatrix}$$

$-1 < \nu_i < 1$

$\nu_a^2 < \frac{1-\nu_i}{2} \cdot \frac{E_a}{E_i}$

Segun Lekhnitskii (1963) y Amadei (1983)

$$\frac{1}{G_a} = \frac{1}{E_a} + \frac{1}{E_i} + \frac{2\nu_a}{E_a}$$